Mixed questions Co-ordinate Geometry – Straight lines

1*. The points P and Q have coordinates $(–1, 6)$ and $(9, 0)$ respectively.

The line l is perpendicular to PQ and passes through the mid-point of PQ.

Find an equation for l, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(Total 5 marks)

2*.

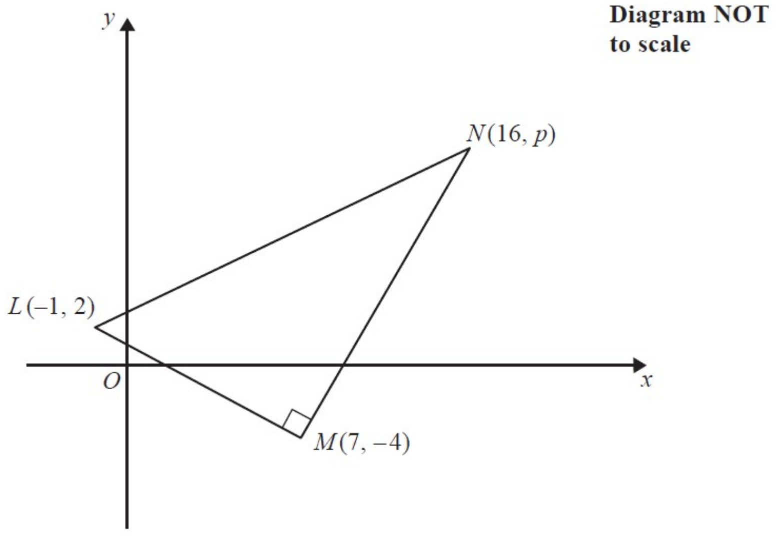


Figure 1

Figure 1 shows a right angled triangle LMN.

The points L and M have coordinates $(–1, 2)$ and $(7, –4)$ respectively.

(a) Find an equation for the straight line passing through the points L and M.

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

Given that the coordinates of point N are $(16, p)$, where p is a constant, and angle $LMN = 90°$,

(b) find the value of p.

(3)

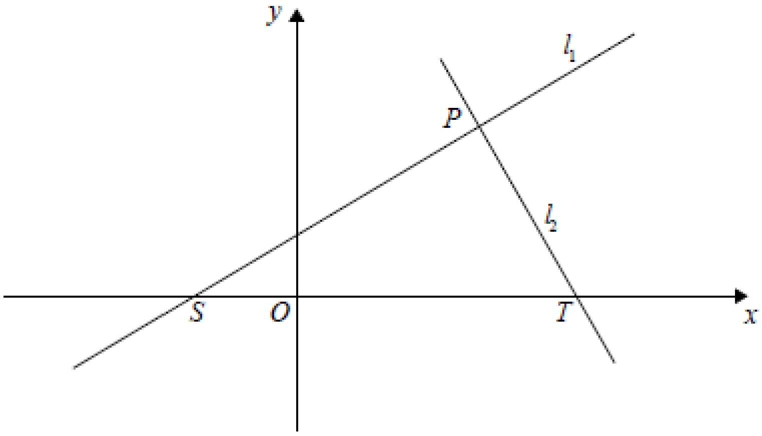
Given that there is a point K such that the points L, M, N, and K form a rectangle,

(c) find the y coordinate of K.

(2)

(Total 9 marks)

3*.



Not to scale

Figure 1

The straight line l_1, shown in Figure 1, has equation $5y = 4x + 10$

The point P with x coordinate 5 lies on l_1

The straight line l_2 is perpendicular to l_1 and passes through P.

(a) Find an equation for l_2, writing your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(4)

The lines l_1 and l_2 cut the x-axis at the points S and T respectively, as shown in Figure 1.

(b) Calculate the area of triangle SPT.

(4)

(Total 8 marks)

Question 4

The vertices of a triangle are the points A (5, 4), B (−5, 8) and C (1, 11).

a Find the equation of the straight line passing through A and B, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

b Find the coordinates of the point M, the mid-point of AC.

6 marks

Question 5

The point A has coordinates (−8, 1) and the point B has coordinates (−4, −5).

a Find the equation of the straight line passing through A and B, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

b Show that the distance of the mid-point of AB from the origin is $k\sqrt{10}$ where k is an integer to be found.

6 marks

Total: 40 marks

Answers

Question	Scheme	Marks
1	Mid-point of PQ is (4, 3)	B1
	PQ: $m = \dfrac{0-6}{9-(-1)}$, $\left(=-\dfrac{3}{5}\right)$	B1
	Gradient perpendicular to PQ $= -\dfrac{1}{m}$ $\left(=\dfrac{5}{3}\right)$	M1
	$y-3=\dfrac{5}{3}(x-4)$	M1
	$5x-3y-11=0$ or $3y-5x+11=0$ or multiples e.g. $10x-6y-22=0$	A1
		(5 marks)
2(a)	Method 1 $gradient = \dfrac{y_1-y_2}{x_1-x_2} = \dfrac{2-(-4)}{-1-7}, = -\dfrac{3}{4}$ Method 2 $\dfrac{y-y_1}{y_2-y_1} = \dfrac{x-x_1}{x_2-x_1}$, so $\dfrac{y-y_1}{6} = \dfrac{x-x_1}{-8}$	M1 A1
	$y-2=-\dfrac{3}{4}(x+1)$ or $y+4=-\dfrac{3}{4}(x-7)$ or $y = their\text{'}-\dfrac{3}{4}\text{'}x+c$	M1
	$\Rightarrow \pm(4y+3x-5)=0$	A1
	Method 3: Substitute $x=-1, y=2$ and $x=7, y=-4$ into $ax+by+c=0$	M1
	$-a+2b+c=0$ and $7a-4b+c=0$	A1
	Solve to obtain $a=3$, $b=4$ and $c=-5$ or multiple of these numbers	M1 A1
		(4)
2(b)	Attempts $gradient\ LM \times gradient\ MN = -1$ so $-\dfrac{3}{4}\times\dfrac{p+4}{16-7}=-1$ or $\dfrac{p+4}{16-7}=\dfrac{4}{3}$ Or $(y+4)=\dfrac{4}{3}(x-7)$ equation with $x=$ 16 substituted	M1
	$p+4=\dfrac{9\times4}{3} \Rightarrow p = \ldots$, $p=8$ So $y =$, $y=8$	M1 A1
		(3)
2(c)	Either $(y=)\ p+6$ or $2+p+4$ Or use 2 perpendicular line equations through L and N and solve for y	M1
	$(y=)\ 14$	A1
		(2)
		(9 marks)

3(a)	Gradient of $l_1 = \frac{4}{5}$ oe	B1
	Point $P = (5, 6)$	B1
	$-\frac{5}{4} = \frac{y - "6"}{x - 5}$ or $y - "6" = -\frac{5}{4}(x - 5)$ or $"6" = -\frac{5}{4}(5) + c \Rightarrow c = ...$	M1
	$5x + 4y - 49 = 0$	A1
		(4)
3(b)	$y = 0 \Rightarrow 5x + 4(0) - 49 = 0 \Rightarrow x = ...$ **or** $y = 0 \Rightarrow 5(0) = 4x + 10 \Rightarrow x = ...$	M1
	$y = 0 \Rightarrow 5x + 4(0) - 49 = 0 \Rightarrow x = ...$ **and** $y = 0 \Rightarrow 5(0) = 4x + 10 \Rightarrow x = ...$	M1
	Method 1: $\frac{1}{2} ST \times "6"$ $\frac{1}{2} \times ('9.8' - '-2.5') \times '6' = ...$ **Method 2:** $\frac{1}{2} SP \times PT$ $\frac{1}{2} \times \sqrt{(5 - '-2.5')^2 + ('6')^2} \times \sqrt{('9.8' - 5)^2 + ('6')^2} = ...$ $\left(= \frac{1}{2} \times \frac{3\sqrt{41}}{2} \times \frac{6\sqrt{41}}{5} \right)$ **Method 3:** 2 Triangles $\frac{1}{2} \times (5 + '2.5') \times '6' + \frac{1}{2} \times ('9.8' - 5) \times '6' = ...$ **Method 4:** Shoelace method $\frac{1}{2}\begin{vmatrix} 5 & 9.8 & -2.5 & 5 \\ 6 & 0 & 0 & 6 \end{vmatrix} = \frac{1}{2}\lvert(0 + 0 - 15) - (58.8 + 0 + 0)\rvert = \frac{1}{2}\lvert -73.8 \rvert = ...$ **Method 5:** Trapezium + 2 triangles $\frac{1}{2} \times ('2.5') \times '2' + \frac{1}{2}("2" + "6") \times 5 + \frac{1}{2} \times ("9.8" - 5') \times '6' = ...$	ddM1
	$= 36.9$	A1
		(4)
		(8 marks)

Question 4 (6 marks)

a grad $= \frac{8-4}{-5-5} = -\frac{2}{5}$

$\therefore y - 4 = -\frac{2}{5}(x - 5)$

$5y - 20 = -2x + 10$

$2x + 5y - 30 = 0$

b $M = \left(\frac{5+1}{2}, \frac{4+11}{2}\right) = (3,\ 7\frac{1}{2})$

Question 5 (6 marks)

a grad = $\frac{-5-1}{-4+8} = -\frac{3}{2}$

$\therefore\ y - 1 = -\frac{3}{2}(x + 8)$

$2y - 2 = -3x - 24$

$3x + 2y + 22 = 0$

b mid-point = $\left(\frac{-8-4}{2}, \frac{1-5}{2}\right) = (-6, -2)$

distance = $\sqrt{6^2 + 2^2} = \sqrt{40}$

$= 2\sqrt{10}$ $[k = 2]$